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Experimental Determination of the Permittivity and Permeability of an Array of Split-Ring Resonators in a X-Band Waveguide

[1]Joaquim P. Leite Neto, [2]Joaquim J. Barroso, [1]Pedro J. Castro, [2]Antonio Tomaz, [2]José Edimar B. Oliveira
[1]Instituto Nacional de Pesquisas Espaciais INPE- São José dos Campos, S.P.
[2]Instituto Tecnológico de Aeronáutica (ITA), 12228-900 São José dos Campos, S.P.

***Abstract* – The permeability and permittivity of an anisotropic metamaterial consisting of a periodic arrangement of resonant rings inserted in a rectangular waveguide excited by magnetic probes are determined experimentally. It is shown that a high transmission is achieved below the cutoff frequency of the empty guide, and depending on the intensity of the coupling between the magnetic probes and resonant rings the transmission system behaves like a balanced CRLH line (Composite Right/Left Handed) or as a purely LH line (Left-Handed). Although sufficient, the condition that the electric permittivity and magnetic permeability be both negative is not a necessary condition to ensure a negative refractive index over a frequency range, a situation in which the phase velocity is oppositely directed to the power flow.**

***Key words* – metamaterials, split-ring resonator, retrieval of constitutive parameters.**

I. INTRODUCTION

Experiments on the propagation and transmission of electromagnetic waves in waveguides loaded with anisotropic magnetic metamaterials (periodic arrangements of split-ring resonators, called SRRs, for example) have enabled the development of subwavelength waveguides, resonators, filters, etc. In the first relevant work in this area, Marques *et al.* [1] showed that in a waveguide loaded with a SRR resonator array [2] a backward wave travels in a narrow frequency range and below the cutoff frequency of the fundamental TE_{10} mode of the empty waveguide. This counterintuitive phenomenon was attributed to the negative permeability introduced by the split-rings inside the waveguide in conjunction with the negative electrical permittivity provided by the waveguide at frequencies below the cutoff frequency. In the experiments of Marques *et al.* [1] and Hrabar *et al.* [3] the subwavelength transmission reached maximum intensity of -20 dB in a pass band centered around the magnetic resonance frequency. The loaded waveguide excitation was made through coaxial probes connected to joints of rectangular wave guides. Other experiments followed to enhance the level of transmission below the cutoff frequency. Carbonell *et al.* [4] used direct coaxial probe inside the evanescent wave guide, with the probe short-circuiting the resonant ring arm (in this case, a square ring) that contained the split.

J. P. Leite. Neto, leite@plasma.inpe.br; J. J. Barroso, barroso@ita.br; P. J. Castro, pedro.castro@inpe.br; A. Tomaz, attomaz@gmail.com; J. E. B. Oliveira, edimar@ita.br.

Transmission losses were minimized to 4 dB. In the present, work we used magnetic probes to excite the SRR-loaded waveguide and studied the effects of weak and strong couplings between the SRR array and the probe through a retrieval procedure of the constitutive parameters, namely the electric permittivity and the magnetic permeability, as well the refractive index and the wave impedance. Detailed knowledge of the constitutive properties of these artificial materials becomes relevant for microwave applications, as for example, antennas, dephasers, and absorbers. All experiments were implemented by loading a rectangular waveguide with a periodic arrangement of six SRRs, with dimensions and properties given in Fig. 1. The array is placed in the plane of symmetry of a X-band waveguide (WR90) of cross-sectional area 2.29 x 1.02 cm^2 and cutoff frequency of 6.55 GHz (Fig. 2).

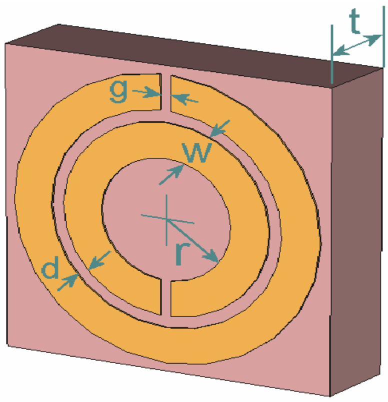

Fig. 1. Schematic of the split-ring resonator (SRR) used in the experiments. Parameters: d = 0.75 mm, w = 0.75 mm (width of the copper rings), t = 1.6mm, r = 2.25 mm, g= 1.13 mm; dielectric constant of the substrate ε = 3.2.

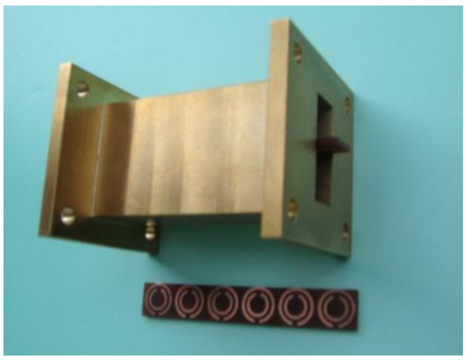

Fig. 2. WR90 waveguide loaded with six SRRs. The lattice constant (distance between the centers of two adjacent rings) is 10. 2 mm.

II. RETRIEVAL OF CONSTITUTIVE PARAMETERS

The properties of the metamaterial inserted in the rectangular waveguide operating in the TE_{10} propagation mode, can be determined by measuring the scattering parameters through a network analyzer. In terms of the semi-infinite reflection and transmission coefficients (Γ, T), the backward and forward complex scattering parameters can be written as [5],[6]:

$$S_{11} = \frac{\Gamma(1-T^2)}{1-\Gamma^2 T^2} \quad ; \qquad S_{21} = \frac{T(1-\Gamma^2)}{1-\Gamma^2 T^2} \tag{1}$$

The reflection coefficient at the input port is given by

$$\Gamma = \kappa \pm \sqrt{\kappa^2 - 1} \tag{2}$$

with

$$\kappa = \frac{S_{11}^2 - S_{21}^2 + 1}{2\, S_{11}}$$

The transmission coefficient along the structure is written as:

$$T = \frac{S_{11}+S_{21}-\Gamma}{1-(S_{11}+S_{21})\Gamma} \tag{3}$$

The impedance of the loaded waveguide retrieved from S-parameters is:

$$Z = \frac{(1+S_{11})^2 - S_{21}^2}{(1-S_{11})^2 - S_{21}^2} \tag{4}$$

In terms of the reflection coefficient, Z is given by:

$$Z = \frac{1+\Gamma}{1-\Gamma} \tag{5}$$

The refractive index n can be retrieved through the transmission coefficient

$$T = e^{-i\,\beta_0 n\, d},\ \beta_0 = \frac{2\pi f}{c} \tag{6}$$

where β_0 is the free-space wave number, f is the operation frequency, and d the length of the SRR array; c is the light velocity in free space. It follows that

$$n = \frac{1}{\beta_0 d}[\pm(\ln T)'' \pm 2\,\pi\, m - j\,(\ln T)'], \tag{7}$$

where $m = 0, 1, 2, 3, \ldots$ is an integer denoting the branch of the logarithmic function, with $(\ln T)'$ and $(\ln T)''$ representing the real and imaginary parts of the logarithmic transmission function, respectively. From (7), a relationship between the guided wavelength, λ_g, and the transmission coefficient is obtained:

$$\frac{1}{\lambda_g} = \frac{j}{2\,\pi\, d} \ln(T) \tag{8}$$

The constitutive parameters can be retrieved from an expression connecting the wave impedance Z and the relative permeability μ_r:

$$Z = \frac{j\omega\mu_0\mu_r}{\beta_z} \tag{9}$$

where

$$\beta_z = j\,\frac{2\,\pi}{\lambda_0}\sqrt{\epsilon_r\mu_r - \mu_r\left(\frac{\lambda_0}{\lambda_c}\right)^2} \tag{10}$$

with μ_0 being the vacuum permeability, λ_0 and λ_c corresponding, respectively, to the free space and empty waveguide cutoff wavelengths, and ϵ_r is the effective permittivity of the metamaterial. The propagation constant β_z takes into account anisotropy of the material, for which the magnetic permeability along the propagation direction is the magnetic permeability of vacuum, which is has been verified in previous experiments [6]. From (5)-(10), it follows that:

$$\mu_r = \frac{\lambda_{0g}}{\lambda_g}\,\frac{1+\Gamma}{1-\Gamma} \tag{11}$$

$$\epsilon_r = \frac{\left(\frac{2\,\pi}{\lambda_g}\right)^2 + \mu_r\,\beta_{0x}^2}{\mu_r\,{\beta_0}^2} \tag{12}$$

where $\beta_{0x} = \frac{\pi}{a}, \beta_0 = \frac{2\,\pi}{\lambda_0}$ and $\lambda_{0g} = \frac{1}{\sqrt{\frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2}}}$ is the guided wavelength for the empty waveguide. The length a is the larger side of the waveguide cross-section (22.85 mm).

III. EXPERIMENTAL RESULTS AND DISCUSSION

First, the waveguide loaded with the SRR array is symmetrically connected on both ends to identical X-band waveguide-to-coaxial adapters, by means of which the input and output signals have been injected and detected by an Agilent N5230C vector network analyzer, as can be seen in Fig. 3. The S_{21} parameter measured by using coaxial probes, displayed in Fig. 4, is at around $f \sim 3.45$ GHz very weak ($S_{21} \sim -45$ dB), which agrees with the results in [1] and [3].

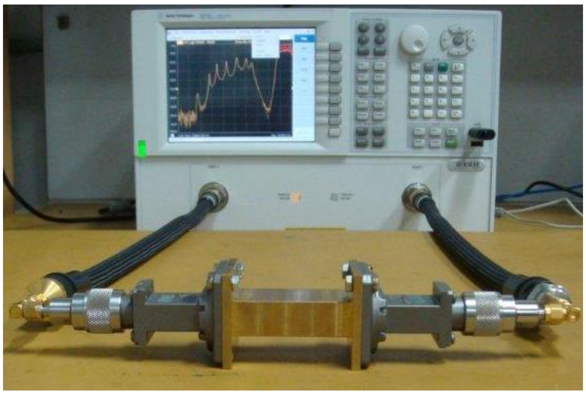

Fig. 3. Set-up to measure the transmission band of the X-band waveguide loaded with the SRR array using waveguide coaxial adapters.

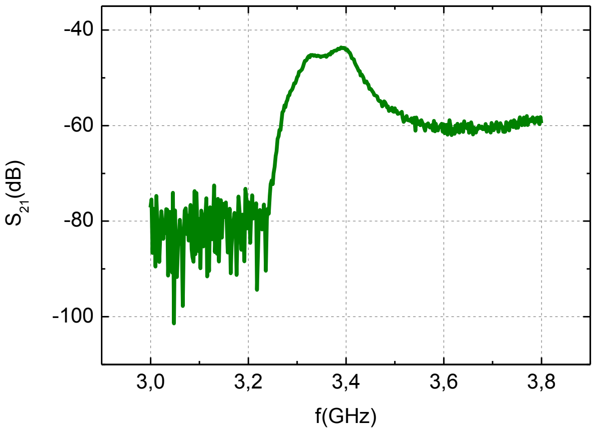

Fig. 4. The transmission band for the waveguide loaded with a periodic arrangement of six SRRs measured with coaxial adapters waveguides.

To enhance the transmission band resulting from the effect of magnetic resonance, rather than using coaxial transitions, a pair of magnetic probes was used to excite and detect the propagating signal, as shown in Fig. 5. The magnitudes and phases of the reflection and transmission coefficients, respectively S_{11} and $S_{21,}$ were obtained for two circumstances: a) weak coupling between the probe and the SRRs; b) strong coupling. They are displayed in Figs. 6 and 7.

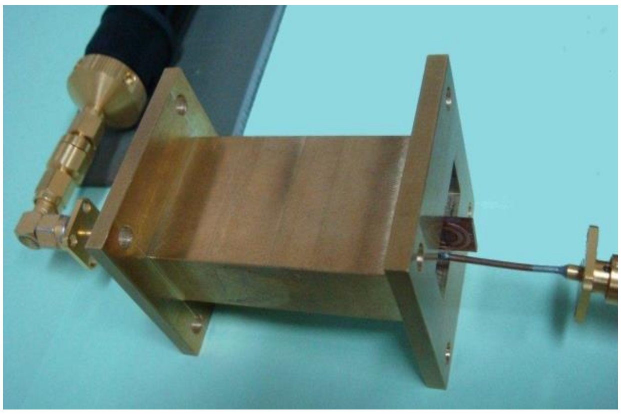

Fig. 5. Experimental setup to single out the magnetic resonance transmission band using two magnetic probes.

In Fig. 6 it is observed that in the case of weak coupling, the maximum transmission intensity (S_{21} ~ 0.22) was obtained at f ~ 3.32 GHz, while in the case of the strong coupling the resonant frequency is slightly reduced (f ~ 3.28 GHz); the transmission band is wider and S_{21} reaches ~ 0.56.

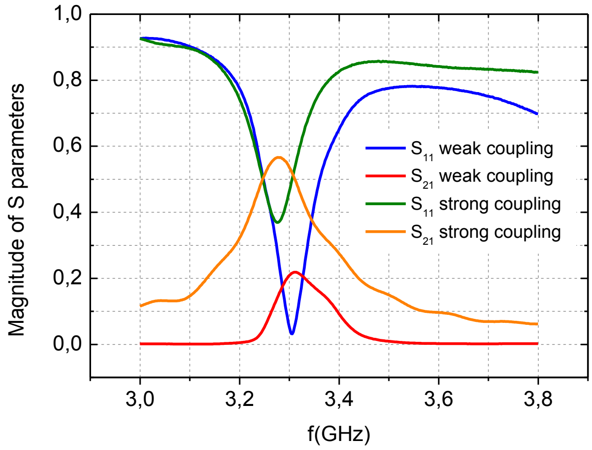

Fig. 6. Magnitudes of reflection (S_{11}) and transmission (S_{21}) scattering parameters. Measurements were made by using magnetic probes as shown in Fig. 5.

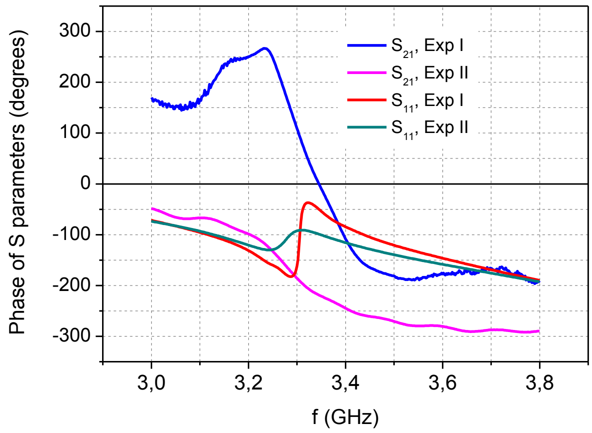

Fig. 7. Unwrapped phases of reflection and transmission coefficients corresponding to the magnetic response of the periodic array of six SRRs inserted into the X-band waveguide.

A dispersion diagram (frequency *vs*. propagation constant) obtained from the S-parameters is presented in Fig. 8. It is demonstrated that, in the case of weak coupling, the wave propagation is backward from 3.0 to 3.32 GHz, since in this frequency range the phase velocity is negative and the group velocity is positive. For higher frequencies (f > 3.32 GHz), both the phase and group velocities are positive and then the propagation is forward. There is no gap between the forward and backward modes, which indicates that this structure is similar to a balanced CRLH transmission line (Composite Right Left Handed) as described in [7] and [8]. The coalescence frequency (phase shift $\beta d = 0$) of the modes is f_0 ~ 3.32 GHz. For the strong-coupling case, however, the dispersion diagram reveals that the balanced CRLH behavior disappears and the guiding system is purely LH (left-handed), that is, the wave is backward in the entire range of frequencies, from 3.0 to 3.8 GHz, i. e., the blue curve lies entirely in the left quadrant.

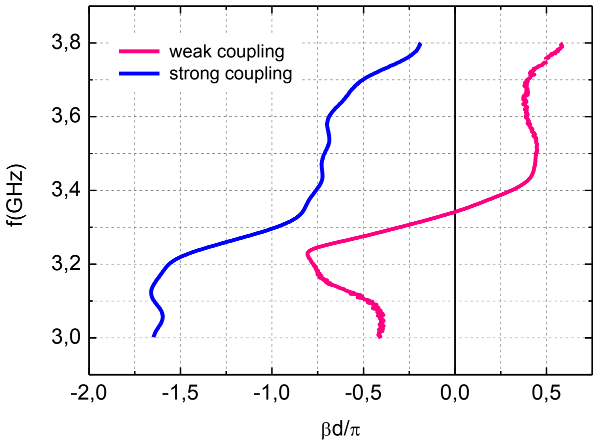

Fig.8. Dispersion diagram for the SRR array inserted in the WR90 waveguide.

The retrieved refractive index is shown in Fig. 9. In the case of strong coupling the real part of the index is negative over all the range of frequencies studied (f ~ 3.0 − 3.8 GHz), indicating negative refraction, with values varying in the range $-1.3 \lesssim n_r \lesssim -0.25$, whereas for weak coupling the real part of the refractive index is negative at frequencies $f \lesssim 3.32$ GHz, taking negative values in the range $-1.25 \lesssim n_r \lesssim 0$; above this frequency, n_r goes up to 0.67. The imaginary parts of the refractive index show that the strong coupling leads to a much lower energy dissipation in the SRR array.

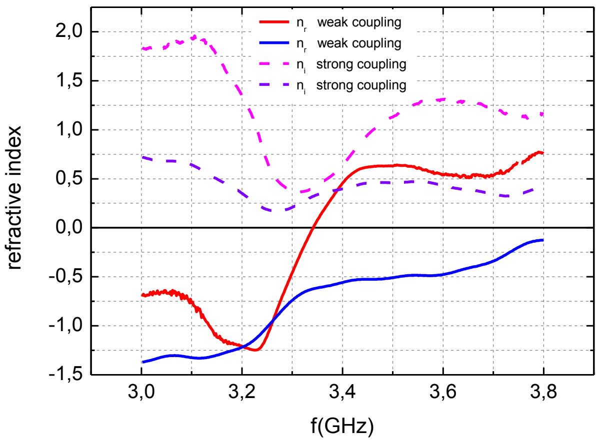

Fig. 9. Refractive index extracted from scattering parameters *S* measured in loaded waveguide. The solid lines refer to the real part n_r and the dotted lines to imaginary part n_i. The minimum of the imaginary part of the refractive index n_i corresponds to strongest transmission.

The retrieved magnetic permeability, displayed in Fig. 10, shows a real negative part in almost all the frequency range of frequencies in both cases of weak and strong couplings. In the case of weak coupling, the magnetic permeability takes on larger negative values in the 3.0-3.2 GHz interval, although in this frequency range the transmission coefficient is negligibly small, as shown in Fig. 6. Note that in the region of higher transmission, between 3.2 and 3.4 GHz in Fig. 6, the permeability values are approximately the same in both coupling cases, thus showing that the effective magnetic permeability is nearly independent of the incident field amplitude, indicating a linear behavior in that the induced magnetic flux density is proportional to the magnetic field strength, namely, $\mu = B/H$.

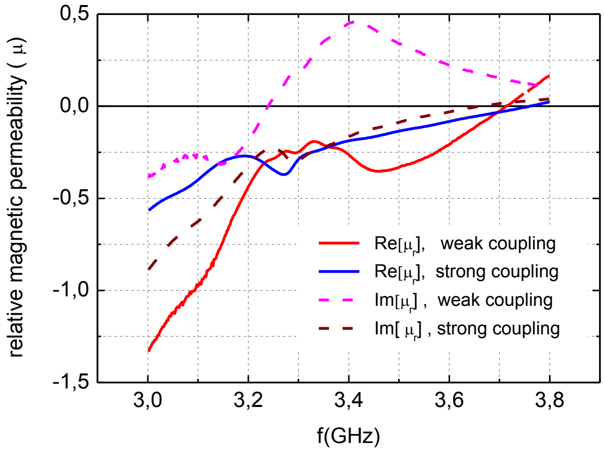

Fig. 10. Magnetic permeability extracted from measured *S*-parameters. Solid lines refer to the real part and dotted lines to the imaginary part.

The electric permittivity ε_r of the SRR periodic array is negative only in the region around the magnetic resonance frequency ($f \sim 3.20 - 3.30\ GHz$) and is more intense in the case of weak coupling, as shown in Fig. 11. In the frequency range $3.30 \leq f \leq 3.65$ GHz we obtained $\varepsilon_r > 0$ both to the strong and weak couplings, while in this frequency range the real part of the magnetic permeability (μ_r) is negative (Fig. 10). Then it would be expected no negative refractive index in the $3.30 \leq f \leq 3.65$ GHz range. But in the strong-coupling case, $n_r < 0$ as demonstrated in Fig. 9. The explanation for this circumstance is discussed in [9] and [10], where it is shown that in lossy materials it is possible that the real part of the refractive index (n_r) be negative without requiring that the real parts of permittivity ε_r and permeability μ_r be simultaneously negative. This happens if the imaginary parts of ε_r and μ_r are large enough. Recalling that in a lossy material $n = n' - j\,n''$, $\varepsilon = \varepsilon' - j\,\varepsilon''$, $\mu = \mu' - j\mu''$, (where the time harmonic variation $\exp(j\omega t)$ is assumed), so that $n = \varepsilon\, Z$ e $Z = \sqrt{\mu/\varepsilon}$, we obtain

$$n' = \varepsilon' Z' - \varepsilon'' Z''$$

$$Z = \sqrt{(\mu'\varepsilon' + \mu''\,\varepsilon''/|\varepsilon|^2) - j\,(\mu''\varepsilon' - \mu'\,\varepsilon'')/\,|\varepsilon|^2}.$$

Therefore it is possible that $n_r < 0$ as long as $\varepsilon'' Z'' > \varepsilon' Z'$. For a quantitative examination, the frequency dependence of the function $\varepsilon'' Z'' - \varepsilon' Z'$ is shown in Fig. 12. Then it is confirmed

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that in the case of weak coupling the wave propagation is backward only at lower frequencies (f < 3.32 GHz), while for the strong-coupling excitation the propagation is backward over all the frequency range studied.

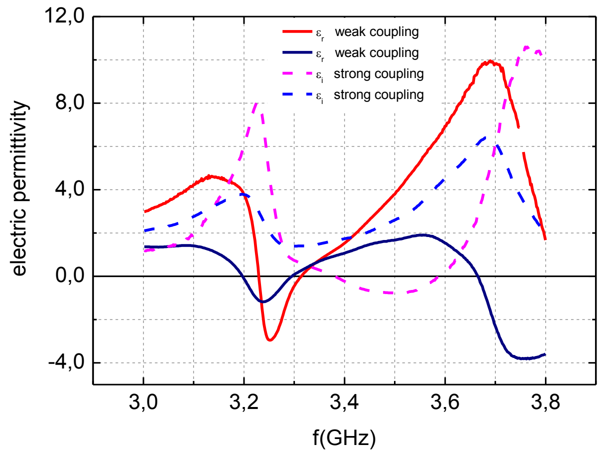

Fig.11. Relative electric permittivity extracted from measured scattering parameters.

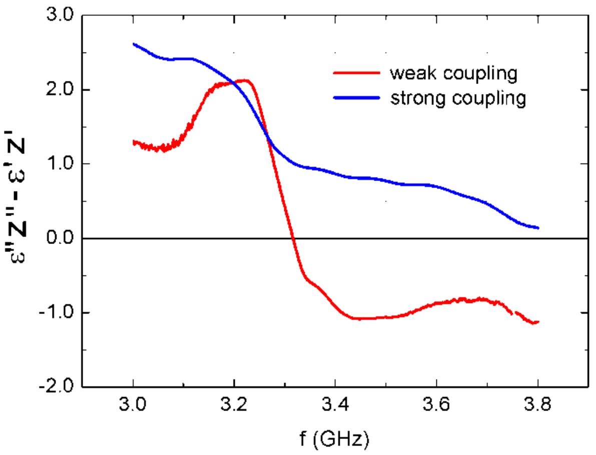

Fig. 12. Condition of Zhou [9] to the backward wave propagation. All regions with positive value refer to the backward propagation.

IV. CONCLUSION

The magnetic permeability and electric permittivity were determined experimentally for an array of split-ring resonators inserted in a X-band rectangular waveguide in the range of frequencies 3.0–3.8 GHz, which contains the magnetic resonance frequency of the rings below the cutoff frequency of the empty waveguide. It was noted that waveguide excitation by magnetic probes has intensified the left-handed (LH) transmission, and depending on the intensity of the waveguide-SRRs coupling, wave propagation can be equivalent to that of a balanced CRLH line (weak coupling) or a purely LH line (strong coupling). We also observed a negative refraction even in the range of frequencies over which the electric permittivity is positive, unlike early studies which required the necessary condition that the permittivity and permeability be simultaneously negative [2], [11].

A reverse procedure (not included in the text) recovers all the measured S-parameters using the retrieved permeability and permittivity, fully validating the accuracy of the constitutive parameters obtained.

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