

Optimized simulation for radio transmission project

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Abstract—The research returns to impulse analysis with signal scattering, developing a prediction model for the range of data in confined areas, with optimization of power points for regions of wide and open access in urban geography. Applying mathematical models, the study presents numerical and comparative results using the Log-Distance and MMSE-Mean Least Squared Error equations, with responses in polynomial and meta-heuristic modeling. The polygons contain geographic coordinates that demonstrate the behavior and dispersion of impulses, returning the quality of the transmitted signal for open and wooded environments, defining points with the best iteration obtained in numerical results of detected electromagnetic signals. The calculated polynomial models are expanded through a system of differential equations, projecting the mathematical range of impulses in the built and confined area.

Keywords— Optimization, signal processing, telecommunications.

I. INTRODUCTION

The study of signals has become increasingly important in the prediction and improvement of telecommunications channels, and in the search for the use of data computation in the set of electronic circuits simulated by mathematical projection. Numerical analysis combined with graphic production allows visualizing the power and transmission range between mobile-phone towers and can predict the best locations for the use of telecommunications devices. Simulation by signal modeling can define how to anticipate construction costs of base stations, seeking to design improvements in mobile technology, as well as in satellite transmissions, considering the great commercial demand for services in communication networks. As research references, the combination of graph study with polynomial analysis for the signal response in discrete time is seen in [1]. The algorithm for Beam and Antenna Selection in mobile communication is observed in [2], to be transformed into an unrestricted optimization problem through the use of Lagrange multipliers. The research undertakes the capture of numerical information through impulses obtained by simple and portable equipment, returning the modeling of two deterministic polynomials, calculated from the numerical interpolation of the Lagrange model. In the second stage of data collection for impulses propagated in free areas containing vegetation, two analyses are implemented both in the convex processing applied by the Graham-Scan method, and which according to [3] solves the convex hull problem, inserting each point of an input set Q in a stack, invalidating points of the same stack that are not vertices of the polygon CH(Q), resulting in the definition of signal technology as a graphic signature. The mathematical modeling anticipates the behavior of the analyzed signals, which is applied in combination with Log-Distance equations and minimum mean square error, and the results are also compared

to the polynomial model proposed in the analysis. The same equipment was used to obtain data, however, as they form a polygonal shape with numerous vertices, the analysis had to be applied graphically and convexly. The estimated data in partial areas can provide a standardized history in the expansion of a telecommunications network. The Tabu Search applied in this work becomes a method that defines the best signal points for the obtained sets of impulses propagated in the surroundings of an open urban area. In [4] the optimized application of four heuristics for the efficient positioning of telecommunications antennas is discussed, in the analysis of cost reduction. The polynomial responses of the numerical data have their models processed in the format of differential functions, where the expanded modeling of the polynomials will graphically fill the non-sampled spaces in the surroundings for the two paths and with the prediction of the power range.

II. PROPAGATION MODELS

The research applied in this work comprises two main aspects of signal propagation in different environments: The reflection of electromagnetic pulses confined in buildings; and the dispersion of vectors emitted in an open environment. For the first case, it is possible to use mathematical models already established in telecommunications, where deterministic behavior is required. The methods can be seen in [5] and are described as: first signal loss model (1), second signal loss model Log-Distance (2) and (3), and the MMSE estimate - least squares mean error (4). The Lagrange polynomial model (5) was added to the study for a new analysis of the approximate behavior of the signal propagation.

The numerical results obtained within a built-up and confined area have their deterministic models predicted through the propagation equations in a closed environment, obtaining the polynomial mathematical functions for the calculation base with signal range prediction.

$$PL_{dB} = -10 \cdot \text{Log} \left(\frac{G_t \cdot G_r \cdot \lambda^2}{(4 \cdot \pi)^2 \cdot d^2} \right) \quad (1)$$

$$\overline{PL}(d) \propto \left(\frac{d}{d_0} \right)^n \quad (2)$$

$$\overline{PL}(dB) = \overline{PL}(d_0) + 10 \cdot n \cdot \text{Log} \left(\frac{d}{d_0} \right) \quad (3)$$

$$J(n) = \sum_{i=1}^k \left(P_i - \hat{P}_i \right)^2 \quad (4)$$

$$P_n(x) = \sum_{j=0}^n y_j \frac{\prod_{i=0, i \neq j}^n (x_1 - x_i)}{\prod_{i=0, i \neq j}^n (x_j - x_i)} \quad (5)$$

A. Numerical analysis in confined environment

Electromagnetic signals can propagate in confined environments, where reflection from different surfaces should cause attenuation in the amplitude of information impulses. It becomes important to generate a deterministic model for a minimal prediction of how the power and amplitude will behave in a given limited space. A mobile device (mini-modem) was used for data collection, with a description of the usage procedure below. The equipment was raised to a height of one meter above the reference ground level, and through the Wifi-Analyzer application average readings of the received power signal were obtained in dBm scale. With an energizing voltage limit of 110 volts and 10 amps, and being fixed and suspended at a height of 25 centimeters from the floor. Data collection transmission was performed by WPA2 PSK modulation for operating frequency at 2.44 Gigahertz. Classical models in telecommunications (1) to (4) generated data by estimation and which were compared to the Lagrange polynomial model seen in (5), deduced from the calculated coefficients.

To define the values estimated by telecommunications equations, the mean of the smallest squared error was initially calculated using the function $J_1(n_1) = 338.0839 n_1^2 + 4581.22 n_1 + 21114.45$, allowing to predict the behavior of the standard deviation of the signal for Path 1, and repeating the same procedure through the function $J_2(n_2) = 338.0699 n_2^2 - 4554.04 n_2 + 20463.21$ referring to Path 2. As a second step, the path loss exponent $n_1 = 6.78$ and $n_2 = 6.74$ are obtained from the relation $\frac{dJ(n)}{dn} = 0$, also obtaining the sample variance $\sigma^2_1 = 1398.74$ with standard deviation $\sigma_1 = 37.40$ for route 1, and the sample variance $\sigma^2_2 = 1281.838$ with standard deviation $\sigma_2 = 35.80$ for route 2. The table I shows the response of the field samples, the estimate of the projection of impulses, and also the calculation of the Log-Normal shading that will represent the proper calculated probabilities of powers (dBm) using the (3) and (4), which are predicted along the two reading paths. The Signal also suffers losses that can be estimated by (1), which include the specification values: From the transmit antenna gain in $G_t = 0.04144$; Receive gain in $G_r = 0.0207$; and the wavelength $\lambda = 0.1231$ m.

The final analysis of the prediction of the distributed signals and powers was given to the two calculated polynomial models, and from the numerical samples of paths 1 and 2. The final result of the calculations of the samples and the coefficients of the polynomials can be seen in the Table I. The polynomials P_1 and P_2 , Table II, formed by Lagrange interpolation, are graphically compared to the sample data with the estimated data, as can be seen in Figure 1 for the first path. Figure 2 also defines the compared data behavior for the second path. This analysis makes it possible to initiate a broader computational study base containing a greater amount of numerical information per sample, extending to different areas and surfaces.

$$P_1 = \sum_{k=9}^1 a_{k-1} x^{k-1}, \quad \text{and} \quad P_2 = \sum_{k=9}^1 b_{k-1} x^{k-1}$$

$$x = \{1, 18 \mid \forall x \in \mathbb{R} : x \geq 0\}$$

TABLE I
POWER RANGE RESPONSE POLYNOMIAL MODELING

Distance (m)	Samples (dBm)		Estimates (dBm)		Signal probability (%)	
	1	2	1	2		dBm
1	0	0	0,00	0,00	79,38	> -30
2	-44	-40	-20,41	-20,26	95,04	> -40
4	-50	-44	-40,82	-40,52	99,34	> -50
6	-42	-47	-52,76	-52,37	99,89	> -60
8	-44	-43	-61,23	-60,78	99,98	> -70
10	-52	-53	-67,80	-67,30	100,00	> -80
12	-58	-47	-73,17	-72,63	100,00	> -90
14	-53	-52	-77,71	-77,13	100,00	> -100
16	-43	-51	-81,64	-81,04	100,00	> -110
18	-57	-60	-85,11	-84,48	100,00	> -120

TABLE II
POLYNOMIAL MODELING

Path P_1			Path P_2		
coefficient		degree	coefficient		degree
+5,32846602	$\cdot 10^{-6}$	x^8	-7,57610987	$\cdot 10^{-5}$	x^8
-4,36352927	$\cdot 10^{-4}$	x^7	+6,06708829	$\cdot 10^{-3}$	x^7
+1,50444878	$\cdot 10^{-2}$	x^6	-2,03786892	$\cdot 10^{-1}$	x^6
-2,86631944	$\cdot 10^{-1}$	x^5	+3,72769097	$\cdot 10^0$	x^5
+3,31971571	$\cdot 10^0$	x^4	-4,03180773	$\cdot 10^1$	x^4
-2,39436632	$\cdot 10^1$	x^3	+2,61605556	$\cdot 10^2$	x^3
+1,03480084	$\cdot 10^2$	x^2	-9,82712252	$\cdot 10^2$	x^2
-2,36611310	$\cdot 10^2$	x^1	+1,92254643	$\cdot 10^3$	x^1
+1,62000000	$\cdot 10^2$	x^0	-1,50900000	$\cdot 10^3$	x^0

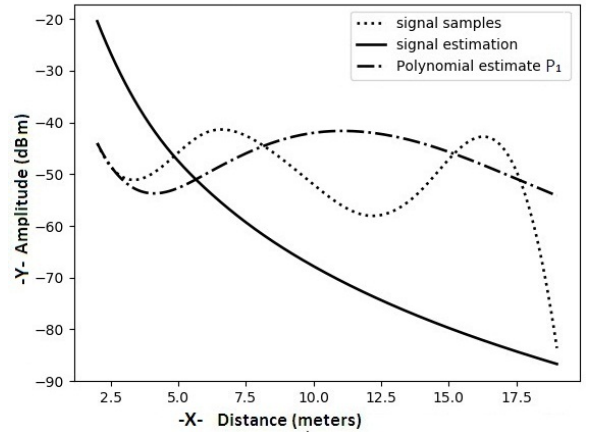


Fig. 1. Set of impulses for the first walk

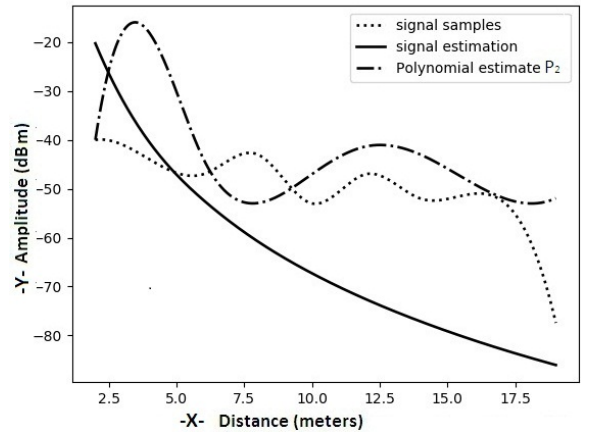


Fig. 2. Set of impulses for the second walk

B. Optimization of signal analysis in open environment

The Tabu Search meta heuristic applied in this research will return a set of optimal solutions for points or vertices with power values of the order of -200 dBm, and in the graphical location of average signal impulses used in mobile devices. In a second analysis, the Graham-Scan method will define which points contained in each polygon of the optimized solution will be convex. The new path will be traced by a set of signal points and used to extract four possible convex vertices, which will define the optimal average value for the power limit and with location prediction. The simulated decision of vertices with coordinates will predict a better positioning of the reception and transmission equipment in an optimized telecommunications network. For computational coding of data samples, the Search-tabu Algorithm 1 is used in conjunction with the analysis convex shape performed by Algorithm 2 and defined by Graham-scan, in which both algorithms can be viewed in the Appendix at the end of this work. For the numerical analysis process, the calculation code written in Python (<https://www.python.org/>) and adapted to the optimization of the force impulse, convex point generating sets are used. As discussed in [6] the convex polygon principle is seen in the identification of *Nodes* for low precision points, and in the location of the radio signal through devices operating in the *Android* system.

The behavior of transmissions in telecommunications is done largely in free space, even if the signals are deflected by different solid obstacles. Through this research and through data collected with simple instrumentation, it was possible to demonstrate the behavior of power and signal pulse technology in free space through two optimization methods, using value collection and modeling analysis that are presented below. Displaced readings were obtained in signal hybrid mode every 02 seconds, using a vehicle with an average speed of 20 km/h, for every 3 meters of distance in each reading. The frequency band was adjusted to LTE/WCDMA/GSM and configured in the Galaxy J2 Prime model cell phone with Android operating system version 6.0.1. The signals are emitted by the mobile operator's base stations, detected along the route and in pairs of readings, round trip, in the geographical area covered by the Federal University of the state of Pará containing the starting point from the Bettina Ferro de Souza university hospital. The G-NetTrack application was chosen to process the signal and also display dynamic readings in real time, where it has low monetary cost with low complexity in obtaining sampled signal values for the power intensity in dBm. The result of the samples returned location coordinates with average signal levels of -130 dBm and for short variations of up to -10 dBm. 3G and 4G technologies were more noticeable along the route, both in closed and open cars. Attenuation values for local vegetation and air humidity and rainfall were not applied.

Real data analysis will require complex electronic instruments, however, it is possible to combine samples of polygonal solutions to the optimized points, allowing to simulate with great mathematical precision the vertices that will have better signal reception capacity. Deterministic equation models can limit predicted results to within their domain sets, but optimization research applied to the collected impulse data can contribute answers beyond mathematical functions, which allowed the return of numerical solutions from

several response points impulse. The solutions in polygonal graphics or numerical data translate the behavior of the average power quality with signal technology, being possible the fixed positioning by geographic coordinates of each graphically determined vertex.

To obtain a response from the first analysis, the convex method Graham-Scan Algorithm 2 was applied to 1204 samples, where it returned four points or vertices containing power and technology values of: (-95 dBm, 3G), (-109 dBm, 3G), (-200 dBm, 4G) and (-18 dBm, 4G) and seen in Figure 3. Continuing the application of the convex method to the 550 samples, returned by optimized tabu search, four points or vertices were also obtained containing power and technology values of: (-95 dBm, 3G), (-109 dBm, 3G), (-135 dBm, 4G) and (-18 dBm, 4G) and seen in Figure 4. With the third and last application of the convex method to the 597 samples, returned by optimized tabu search, four points or vertices were also obtained containing power and technology values of: (-95 dBm, 3G), (-109 dBm, 3G), (-18 dBm, 4G) and (-200 dBm, 4G) and seen in Figure 5. The two central points of both analyzes had an average power around the -200 dBm range,

with a signal level for reception and transmission power at -120 dBm of a mobile-phone device with 3G and 4G technologies detectable. From the vertices of the original polygon of 1024 samples, a convex polygonal Figure 6 was drawn to obtain a graphical response of power behavior and technology in comparison to the two initial analyses. The combined methods in numerical simulation proved to be practical and of low cost, in the study of electromagnetic signal samples through simple equipment. Table III demonstrates that the data variance of the three polygonals Fig. 3, Fig. 4 and Fig. 5 remain close due to the low standard deviation.

Average powers return reception values around -120 dBm, returning two of the four values from the convex point sample set within the range of 0 to -98 dBm for 4G technology. With the graphical response presented, the designer will develop technical applications in electronic transmission equipment to the programmable simulation set, with the best positioning for sending and reaching signal impulses through the optimized vertices for power and technology values.

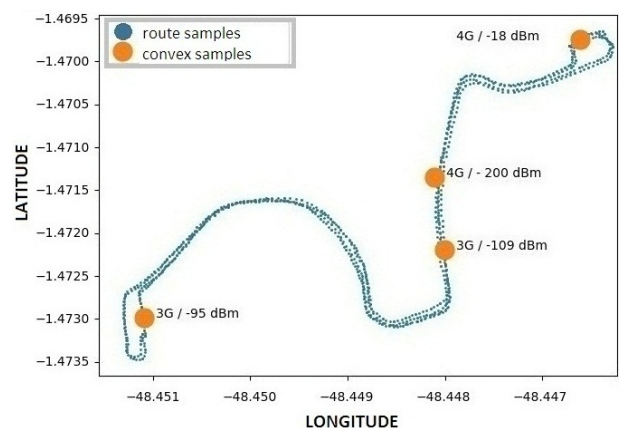


Fig. 3. Polygonal full path

III. SIGNAL PROJECTION

For the various mathematical models that enable propagation predictions of the transmitted signal by ordering possible scattering with reflections, new methods in probability and optimization can add signal points that go beyond deterministic sets. Functions defined by polynomials in this work brought signal impulse responses into the set of samples obtained. However, a differential linear combination was defined for the polynomials P_1 and P_2 with the aim of forcing a new mathematical equation to predict the spread of signal pulses beyond the sampling limits.

In [7] the Zernike polynomial has its analysis applied to phase control for the mobile phone signal, together with the optimized study for improvements in tuning in telecommunications. With cost compensation [8] studies Affine function arithmetic in calculation blocks, considering uncertainties and rounding errors and data truncation. With the analytical study of randomness in differential equations [9] compares mean stochastic solutions with deterministic mean values. Using linear systems of ordinary differential equations [10] applies to the resolution of homogeneous models that are created for a fundamental model of epidemiology.

The notations $\mathbf{y}_{1h}(x)$ and $\mathbf{y}_{2h}(x)$ replace the polynomial equations 1 and 2, where the calculations are developed using the basis of the (6) and (7) differential models of homogeneous mathematical functions. As a segment of continuity to the general equation model $\mathbf{y}_1(\mathbf{x})$ and $\mathbf{y}_2(x)$, the notations $\mathbf{y}_{1p}(x)$ and $\mathbf{y}_{2p}(x)$ were applied.

$$\mathbf{y}'(x) = \frac{d\mathbf{y}}{dx}; \mathbf{y}''(x) = \frac{d^2\mathbf{y}}{dx^2}; \mathbf{y}^{(3)}(x) = \frac{d^3\mathbf{y}}{dx^3}; \mathbf{y}^{(n)}(x) = \frac{d^n\mathbf{y}}{dx^n}$$

$$\mathbf{y}^n(x) \mid \forall n \in \mathbb{N} : n \geq 1 \mid \forall x \in \mathbb{R} : x \geq 1\}$$

$$\left(\sum_{n=9}^1 b_{n-1} \frac{d^{(n-1)}\mathbf{y}_1(x)}{dx^{(n-1)}} \right) = 0 \quad (6)$$

$$\left(\sum_{n=9}^1 b_{n-1} \frac{d^{(n-1)}\mathbf{y}_2(x)}{dx^{(n-1)}} \right) = 0 \quad (7)$$

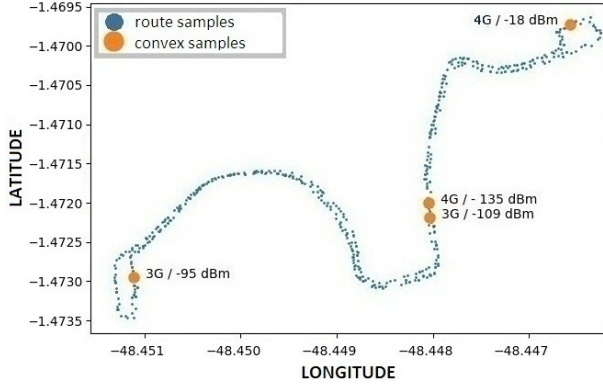


Fig. 4. Second polygonal optimized

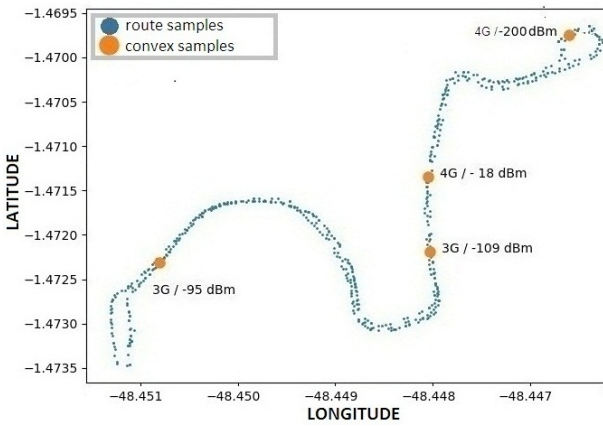


Fig. 5. Third polygonal optimized

TABLE III
DATA ANALYSIS

Statistic	Original Polygonal	Optimization		Unit
		First solution	Second solution	
Average	-111,93	-112,72	-111,7	dBm
Variance	1.622,26	1.532,33	1.605,86	dBm ²
Standard deviation	40,28	39,15	40,07	dBm
Average	6,412·10 ⁻¹²	5,346·10 ⁻¹²	6,760·10 ⁻¹²	mw

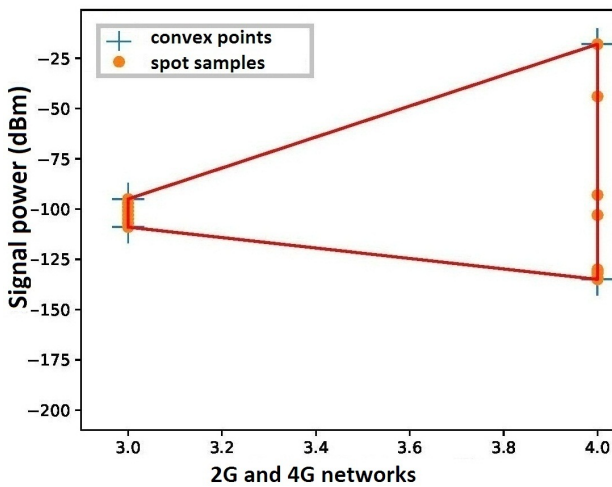


Fig. 6. Convex analysis points of optimized original polygonal

A. Differential projection of polynomials

With measured signal estimation, it is possible to graphically estimate through mathematical models how far the samples can reach in distance, and the power must be close to the gain and coherence of information received. The steps that follow in modeling deduce the homogenous equation with a differential basis for polynomials 1 and 3, and (8) and (14) assume the functions $y_{1h}(x)$ and $y_{2h}(x)$ and that x corresponds to the distance of the projected signal.

The calculation method used in (9) and (15) are responsible for defining the parameters A, B, C, D, E, F, G and H to constant values, returning a matrix of 8 rows by 8 columns and a matrix of 6 rows by 6 columns, for the initial impulse response data in dBm and seen in Table I.

The modeling of the complete equations $y_1(x)$ $y_2(x)$, will gather the sum of the homogeneous equations with the particular equations $y_{1p}(x)$ and $y_{2p}(x)$, using the mathematical functions J_1 and J_2 through their parameters K and R in difference equations where the result is presented in (13) and (19).

First model deduced.

$$\begin{aligned} \mathbf{y}_{1h}(\mathbf{x}) = & + \mathbf{A}_1 e^{1,09x} + \mathbf{B}_1 e^{20,9x} + \mathbf{C}_1 e^{4,97x} \cos(4,37x) \\ & + \mathbf{D}_1 e^{4,97x} \sin(4,37x) + \mathbf{E}_1 e^{8,10x} \cos(6,18x) \\ & + \mathbf{F}_1 e^{8,10x} \sin(6,18x) + \mathbf{G}_1 e^{16,84x} \cos(2,65x) \\ & + \mathbf{H}_1 e^{16,84x} \sin(2,65x) = 0 \end{aligned} \quad (8)$$

$$\begin{cases} \mathbf{y}_{1h}(0) = A_1 f_1(x) + \dots + H_1 f_8(x) = 0 \\ \mathbf{y}'_{1h}(0) = A_1 f'_1(x) + \dots + H_1 f'_8(x) = -44 \\ \vdots \\ \mathbf{y}^6_{1h}(0) = A_1 f_1^6(x) + \dots + H_1 f_8^6(x) = -58 \\ \mathbf{y}^7_{1h}(0) = A_1 f_1^7(x) + \dots + H_1 f_8^7(x) = -53 \end{cases} \quad (9)$$

Elaboration for the final differential equation $y_1(x)$, containing $J_1(x)$ (10) for the modeling steps in (10), (11) and (12).

$$\mathbf{J}_1(x) = \mathbf{J}_1(n_1) = 338.0839n_1^2 + 4581.22n_1 + 21114.45 \quad (10)$$

$$\mathbf{y}_{1p}(x) = \frac{d^{(7)}\mathbf{J}_1}{dx^{(7)}} + \dots + \frac{d'\mathbf{J}_1}{dx'} + \mathbf{J}_1 = \mathbf{K}_1 x^2 + \mathbf{K}_2 x + \mathbf{K}_3 \quad (11)$$

$$\mathbf{y}_{1p}(x) = 338,08x^2 - 3905,05x + 24343,33 \quad (12)$$

$$\mathbf{y}_1(\mathbf{x}) = \mathbf{y}_{1h}(\mathbf{x}) + \mathbf{y}_{1p}(\mathbf{x}) \quad (13)$$

Second model deduced.

$$\begin{aligned} \mathbf{y}_{2h}(\mathbf{x}) = & + \mathbf{A}_2 e^{2,53x} \cos(0,51x) + \mathbf{B}_2 e^{2,53x} \sin(0,51x) \\ & + \mathbf{C}_2 e^{7,06x} \cos(2,40x) + \mathbf{D}_2 e^{7,06x} \sin(2,40x) \\ & + \mathbf{E}_2 e^{12,96x} \cos(2,56x) + \mathbf{F}_2 e^{12,96x} \sin(2,56x) = 0 \end{aligned} \quad (14)$$

$$\begin{cases} \mathbf{y}_{2h}(0) = A_2 g_1(x) + \dots + F_2 g_6(x) = 0 \\ \mathbf{y}'_{2h}(0) = A_2 g'_1(x) + \dots + F_2 g'_6(x) = -40 \\ \vdots \\ \mathbf{y}^4_{2h}(0) = A_2 g_1^4(x) + \dots + F_2 g_6^4(x) = -43 \\ \mathbf{y}^5_{2h}(0) = A_2 g_1^5(x) + \dots + F_2 g_6^5(x) = -53 \end{cases} \quad (15)$$

Elaboration for the final differential equation $y_2(x)$, containing $J_2(x)$ (10) for the modeling steps in (16), (17) and (18).

$$\mathbf{J}_2(x) = \mathbf{J}_2(n_2) = 338.0699n_2^2 + 4554.04n_2 + 20463.21 \quad (16)$$

$$\mathbf{y}_{2p}(x) = \frac{d^{(5)}\mathbf{J}_2}{dx^{(5)}} + \dots + \frac{d'\mathbf{J}_2}{dx'} + \mathbf{J}_2 = \mathbf{R}_1 x^2 + \mathbf{R}_2 x + \mathbf{R}_3 \quad (17)$$

$$\mathbf{y}_{2p}(x) = 338,07x^2 - 5230,14x + 25017,21 \quad (18)$$

$$\mathbf{y}_2(\mathbf{x}) = \mathbf{y}_{2h}(\mathbf{x}) + \mathbf{y}_{2p}(\mathbf{x}) \quad (19)$$

B. projection graphics

With the graphical result presented, Figure 7 returns several pulses estimated throughout an area defined by nine samples in four projections where the signal can be converted into stationary, with corresponding values in dBm.

As a comparison to non-stationary signals, Figure 8 presents signal response still in dBm in which the combined model of Polygonal 2 and 1 with independent variables in x and y is analyzed, seeking to project signals in orthogonal lines. The inverted procedure is proposed in Figure 9 by projecting the combination of Polygons 2 and 1 with for the independent variables in y and x .

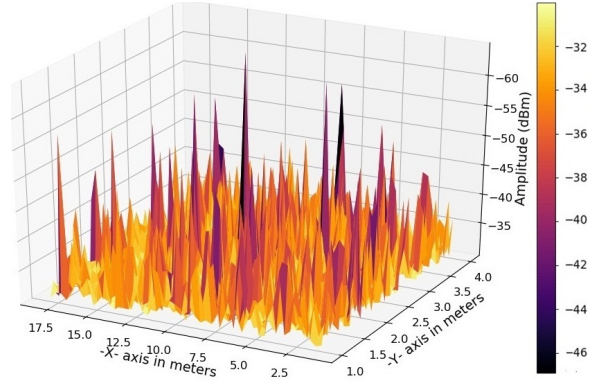


Fig. 7. polynomial $\mathbf{P}_1(\mathbf{x}) \cdot \mathbf{y}_1(\mathbf{y})$ stationary signal

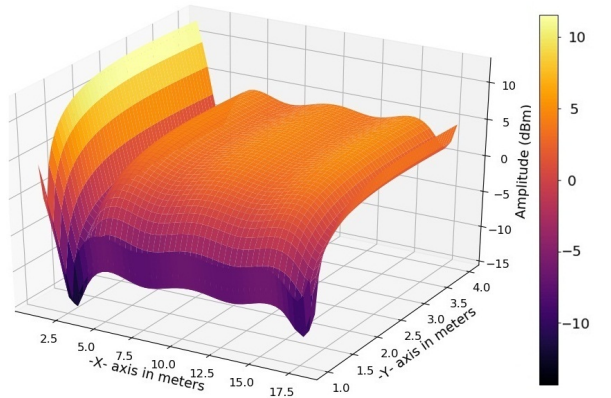


Fig. 8. polynomial $\mathbf{P}_2(\mathbf{x}) \cdot \text{polynomial } \mathbf{P}_1(\mathbf{y})$, non-stationary signal.

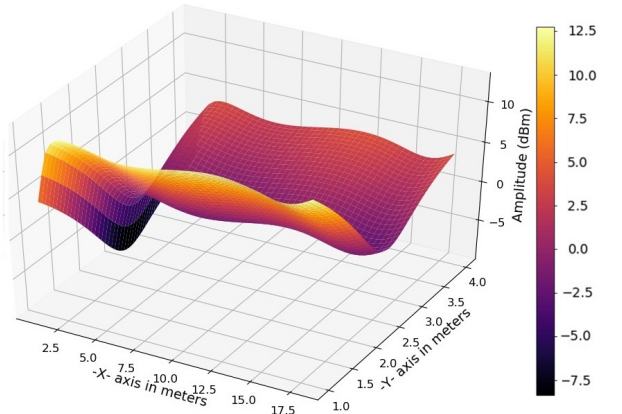


Fig. 9. polynomial $\mathbf{P}_2(\mathbf{y}) \cdot \text{polynomial } \mathbf{P}_1(\mathbf{x})$, non-stationary signal.

The numerical feedback obtained by polynomial data processing demonstrates that the simulation of differential equations can provide a prediction for the behavior of transmission coverage in telecommunications, using a set of impulse samples in power signals in dBm. The three graphical presentations, combining difference equations (13) and (19), provide the possibility of adjustments to the proposed mathematical models and expanding the scope of propagated electromagnetic information.

IV. FINAL RESULT AND APPLICATION

The use of simple electronic equipment limited the number of signal reception parameters, which influenced the values of impulse response rates obtained for analysis. A fact that would not occur if industrial professional equipment with better processing were used in a greater number of devices. The simulation used allowed the analysis of behavior in the processing of large amounts of data, allowing to go beyond the limitation of parameters and equipment, using computational programming applied to the numerical method with mathematical models of transmission for two different environments. With the result demonstrated by iteration of impulses and applied to the projection by graphical simulation, it is possible to predict the range and quality of signal transmission power for both confined and open environments, optimizing numerous data obtained in urban areas with free access. With the Meta-heuristic being applied to the original polygonal path, it becomes possible to expand the classical equation models in telecommunications through tracing of vertex paths. Being able to define the best reception and transmission positions for mobile devices, in the prediction of signals in demarcating geographic points for transmission antennas with the expansion in telecommunications around the world, and in the requirements of field test measurements.

V. CONCLUSION

The result of the signal analysis presented confirms the ability in numerical simulation, as the basis of a low complexity embedded system project, proving that the efficiency of mathematical modeling applied in already established telecommunications equations, can be incorporated in the computational language coding. The limitation on data parameters and electronic equipment was compensated with a large collection of modeled numerical values in addition to defined equations, where optimization can evaluate non-deterministic behavior with graphical pulse response of transmitted electromagnetic signals. This work makes it possible for students, researchers and the telecommunications industry itself to invest in simple and low-cost equipment to predict the anticipated behavior of transmission signals in mobile telephony. The technological expansion of the research can have several points of analysis with more sophisticated industrial electronic equipment, which process a large amount of data using a greater number of parameters.

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APPENDIX

Algorithm 1: TABU SEARCH

```

1 Generate an initial solution  $S$ , and do  $S^* := S$ 
2 Initialize Tabu List ( $T$ ) and counters  $p$  and  $q$ 
3 begin
4   While  $p \neq p_{max}$  and  $q \neq q_{max}$  Do it
5     begin
6       Select the best neighbor  $S' \in N(S) \setminus T$ 
7       Select the best neighbor  $S'' \in N(S) \cap T$ 
8       begin
9         If  $f(S'') < f(S')$  and  $f(S'') < f(S^*)$  then
10           $S' \leftarrow S''$ 
11          If  $f(S') < f(S^*)$  then
12             $S^* \leftarrow S'$ 
13             $q \leftarrow 0$ 
14          If  $f(S') < f(S)$  then
15            put the inverse move  $(S', S)$  in list  $T$  and update  $T$ 
16             $S \leftarrow S'$ 
17             $p \leftarrow p + 1$ 
18             $q \leftarrow q + 1$ 
19          end
20        end
21      end
22 Exit with the solution  $S^*$ , for an optimized set of filters digital signals in a
    signal transmission network.

```

Algorithm 2: GRAHAM-SCAN(Q)

```

1 begin
2   Let  $p_0$  be the point in  $Q$  with minimum coordinate  $y$  or such a point
3   that is most to the left in case of a tie
4   Let  $(p_1, p_2, \dots, p_n)$  be the remaining points in  $Q$ , sorted by polar angle
5   in counterclockwise order around  $p_0$ , where if hover more
6   than a point with the same angle, all points are removed, except
7   the farthest from  $p_0$ 
8   Let  $S$  be an empty stack
9   Move( $p_0, S$ )
10  Move( $p_1, S$ )
11  if  $m \geq 2$  Move( $p_2, S$ ) then
12    for  $i = 3$  to  $m$  do
13      while the angle formed by the points Near Top( $S$ ), Top( $S$ ) and
14       $p_i$  curve does not turn left do
15        | Stacks)
16      end
17      Move( $p_i, S$ )
18    end
19  end
20 end
21 return  $S$ 

```
